

Non-stationary time series attribution for heatwaves over Europe

Pascal Meurer, Sebastian Buschow, Svenja Szemkus, and Petra Friederichs

Institute of Geosciences, Meteorology Section, University of Bonn

Idea:

We attribute complete heatwave time series using a non-stationary Markov model for temporal extremal dependence.

Research questions

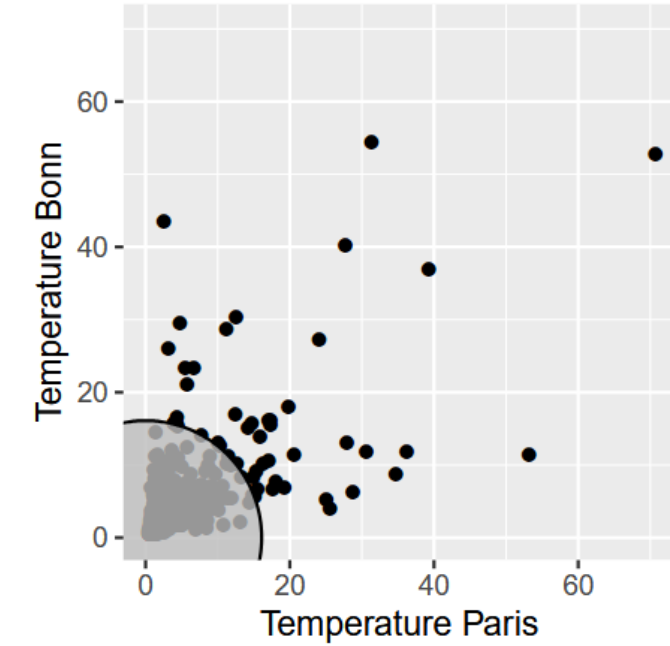
1. Can [changes in] the temporal evolution of a heatwave be attributed to anthropogenic emissions?

2. Is there a climate change signal in the tail behaviour of temperature extremes beyond a general shift in the temperature distribution?

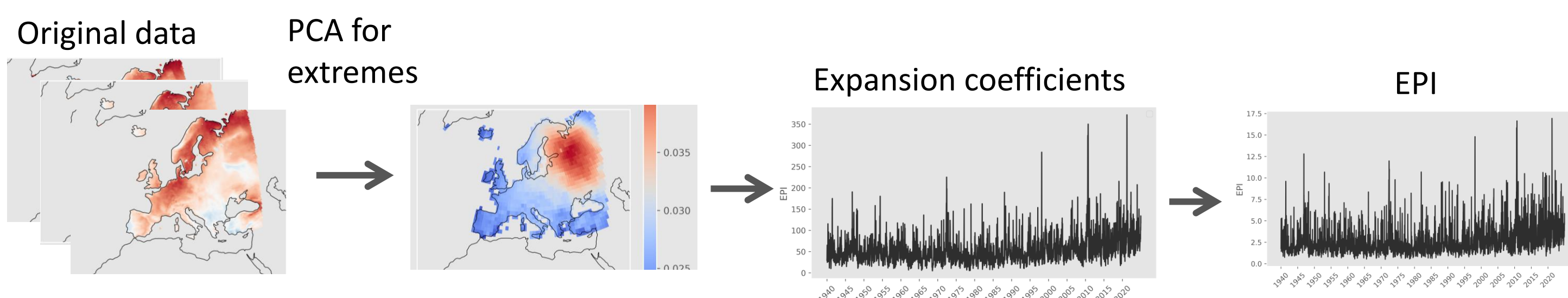
Data adaptive method

Tail Pairwise Dependency Matrix (TPDM) based on threshold exceedances of standardised data (Cooley and Thibaud, 2019)

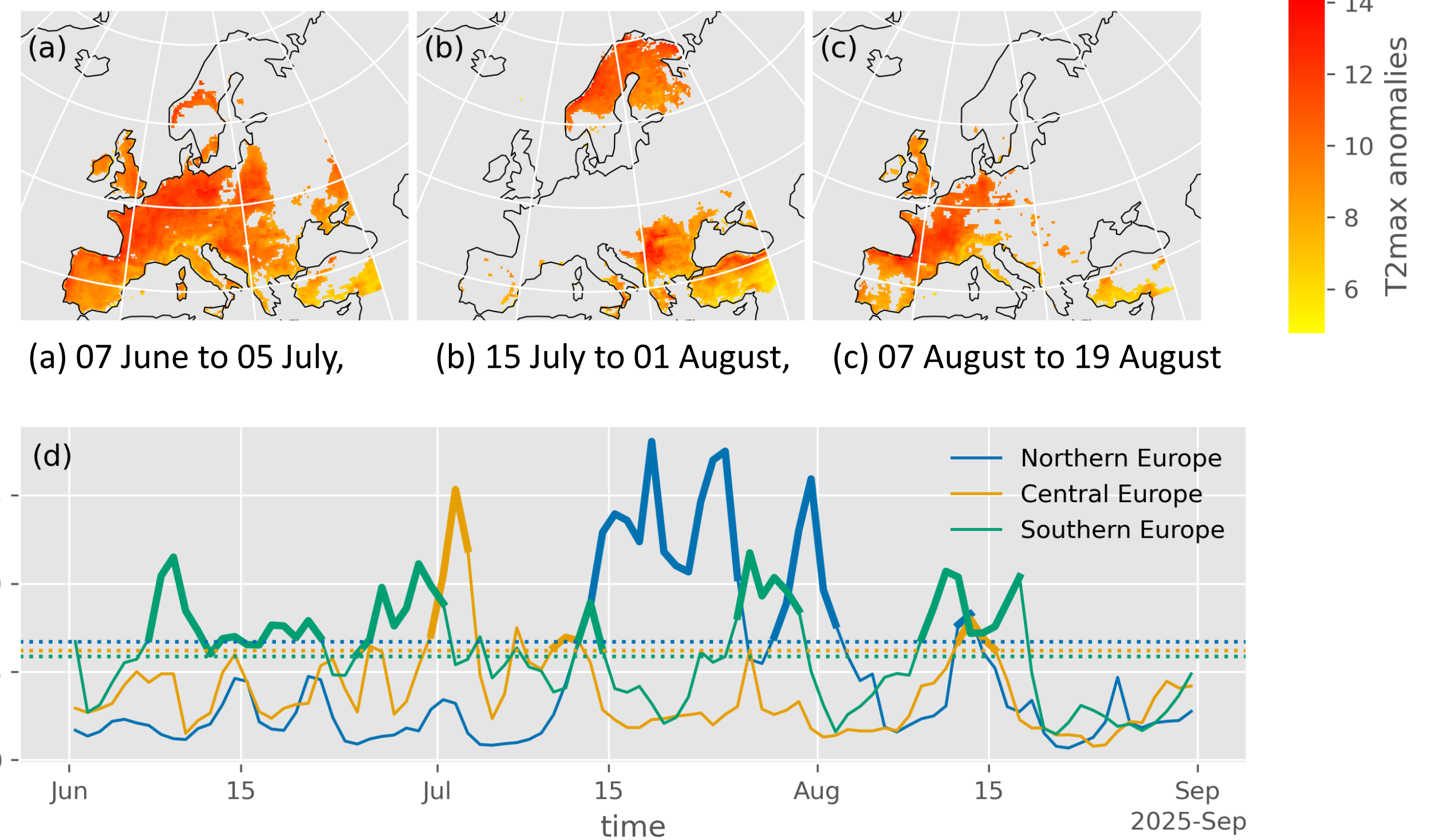
$$\hat{\sigma}_{ij} = 2n_{exc}^{-1} \sum_{t=1}^n w_{t,i}^x w_{t,j}^y \mathbb{I}(r_{t,ij} > r_{0,ij})$$



Extremal Pattern Index (EPI) (Szemkus & Friederichs 2024): Compact description of multivariate weather extremes



Example: summer of 2025



Attribution of extreme events

„standard“ attribution:

$$PN = \mathbb{P}(\neg X | do(m_0), m_1, X)$$

with probability ratio

$$PR = \frac{p_1}{p_0}$$

Probability of event in factual world
Probability of event in counterfactual world

time series attribution:

$$LR(d) = \frac{l(d|m_1)}{l(d|m_0)}$$

Use 1.st order Markov assumption

PR becomes likelihood ratio (LR) with time series d , scenarios m_0 (HIST-NAT) and m_1 (HIST)

$$l(d; \theta) = l(d_1, \dots, d_n; \theta) = l(d_1; \theta_1) \prod_{i=2}^n l(d_i | d_{i-1}; \theta_1, \theta_2)$$

θ_1 : marginal parameters, θ_2 : dependence parameters

Question that arises:

How to model $l(d; \theta)$ for each scenario m_i in order to calculate the likelihood ratio?

Multivariate extreme value theory

Multivariate extreme value theory is based on max-domain of attraction condition

- $F(d_1, d_2) \approx \exp(-l(v_1, v_2))$ with $v_j = \phi_j(1 + \xi \frac{d_j - u_j}{\sigma_j})^{-1/\xi_j}$
- $l(v_1, v_2) = (v_1^{1/\alpha} + v_2^{1/\alpha})^\alpha$, α as the dependence parameter (Logistic model)
- $\alpha \rightarrow 0$: complete dependence, $\alpha = 1$: independence

1. Constant threshold

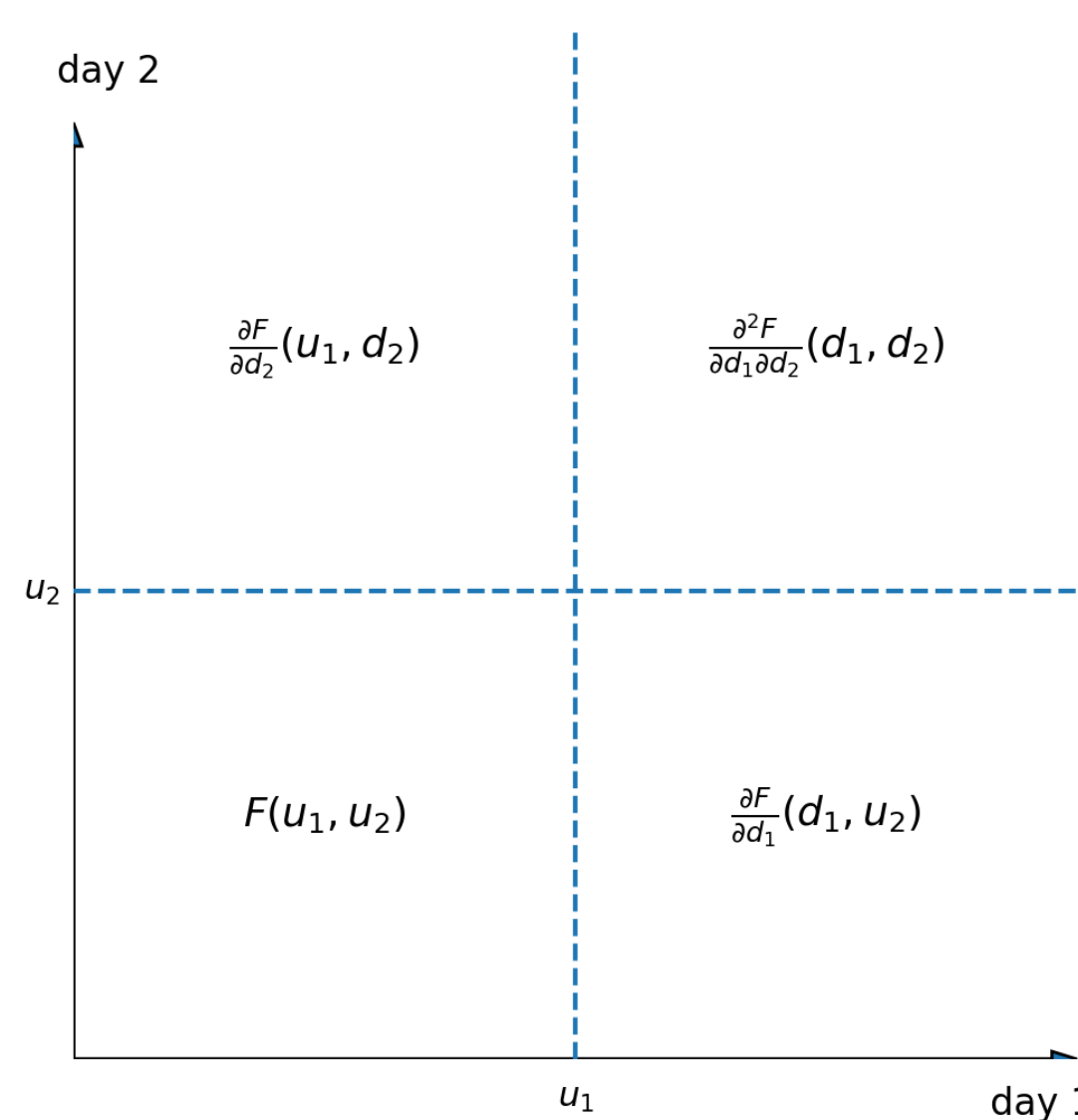
$$\begin{aligned} \logit(\phi_t) &= \beta_u^T g_t, \\ u &= \text{const.}, \\ \sigma_t &= \exp(\zeta_\sigma^T g_t), \\ \xi_t &= \zeta_\xi^T g_t, \\ \alpha_t &= (1 + \exp(-\zeta_\alpha^T g_t))^{-1} \end{aligned}$$

2. Variable threshold

$$\begin{aligned} \phi &= 0.05, \\ u_t &= \beta_u^T g_t \text{ (quantile reg.)}, \\ \sigma_t &= \exp(\zeta_\sigma^T g_t), \\ \xi_t &= \zeta_\xi^T g_t, \\ \alpha_t &= (1 + \exp(-\zeta_\alpha^T g_t))^{-1} \end{aligned}$$

Legendre polynomials $g_t = g_0(t), \dots, g_5(t)$ as covariates with t as time

Censored likelihood model



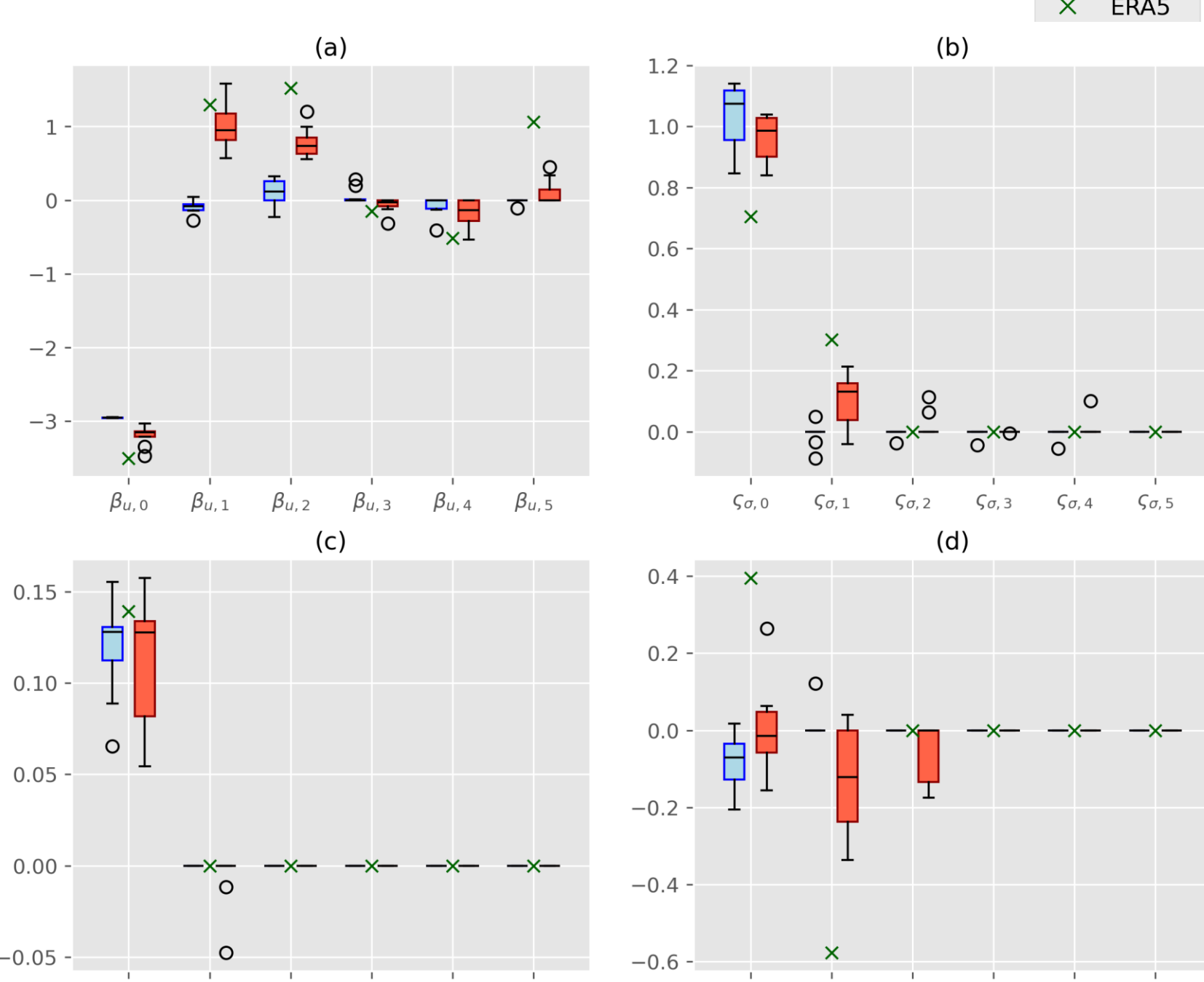
$$l(d_1, d_2; \theta_1, \theta_2) \propto \begin{cases} F(u_1, u_2) & \text{if } d_1 \leq u_1, d_2 \leq u_2 \\ \frac{\partial F}{\partial d_1}(u_1, u_2) & \text{if } d_1 > u_1, d_2 \leq u_2 \\ \frac{\partial F}{\partial d_2}(u_1, u_2) & \text{if } d_1 \leq u_1, d_2 > u_2 \\ \frac{\partial^2 F}{\partial d_1 \partial d_2}(u_1, u_2) & \text{if } d_1 > u_1, d_2 > u_2 \end{cases}$$

We want to estimate θ for each scenario
→ include non-stationarity in parameter modeling

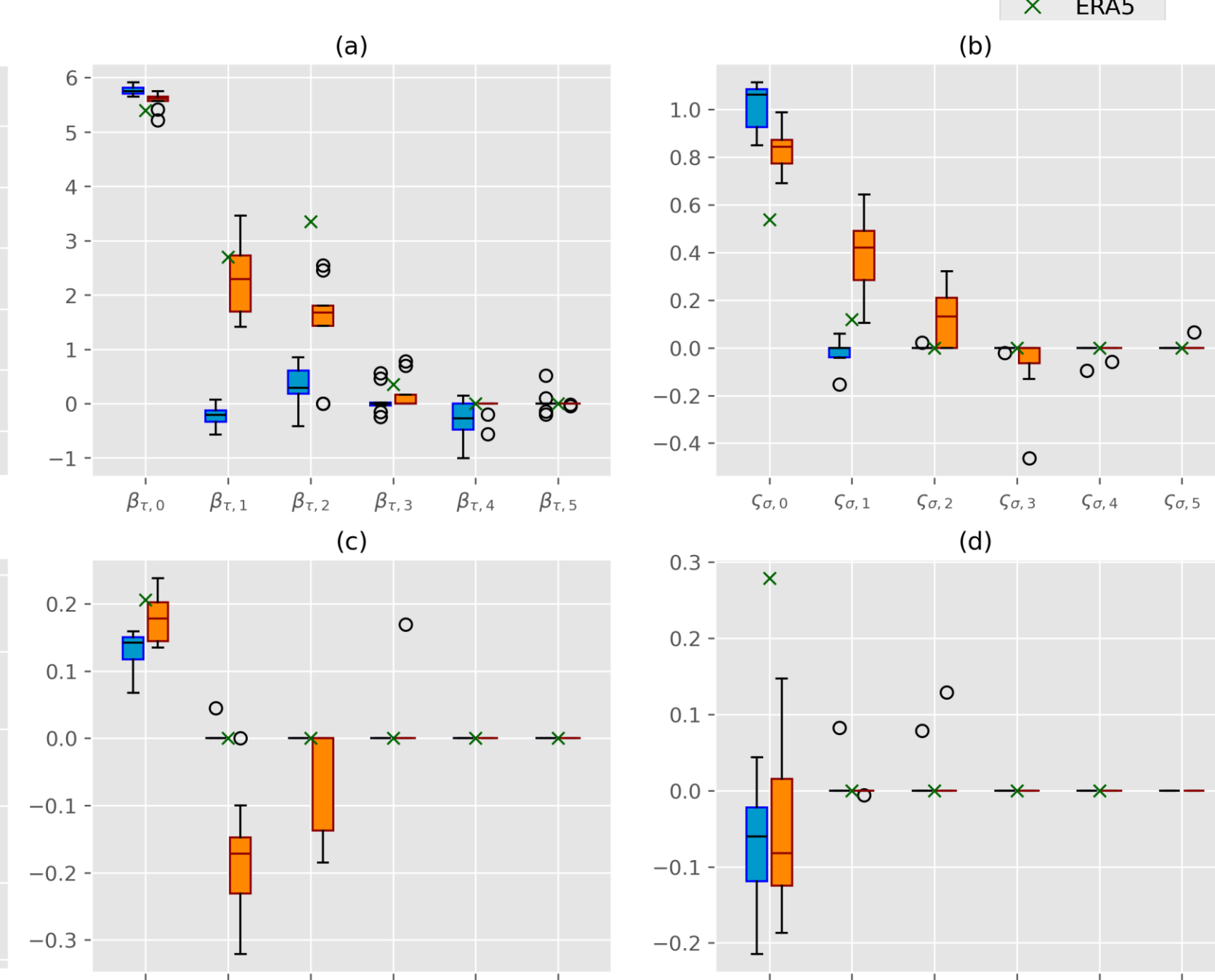
$$\theta = (\sigma, \xi, \alpha)$$

Results

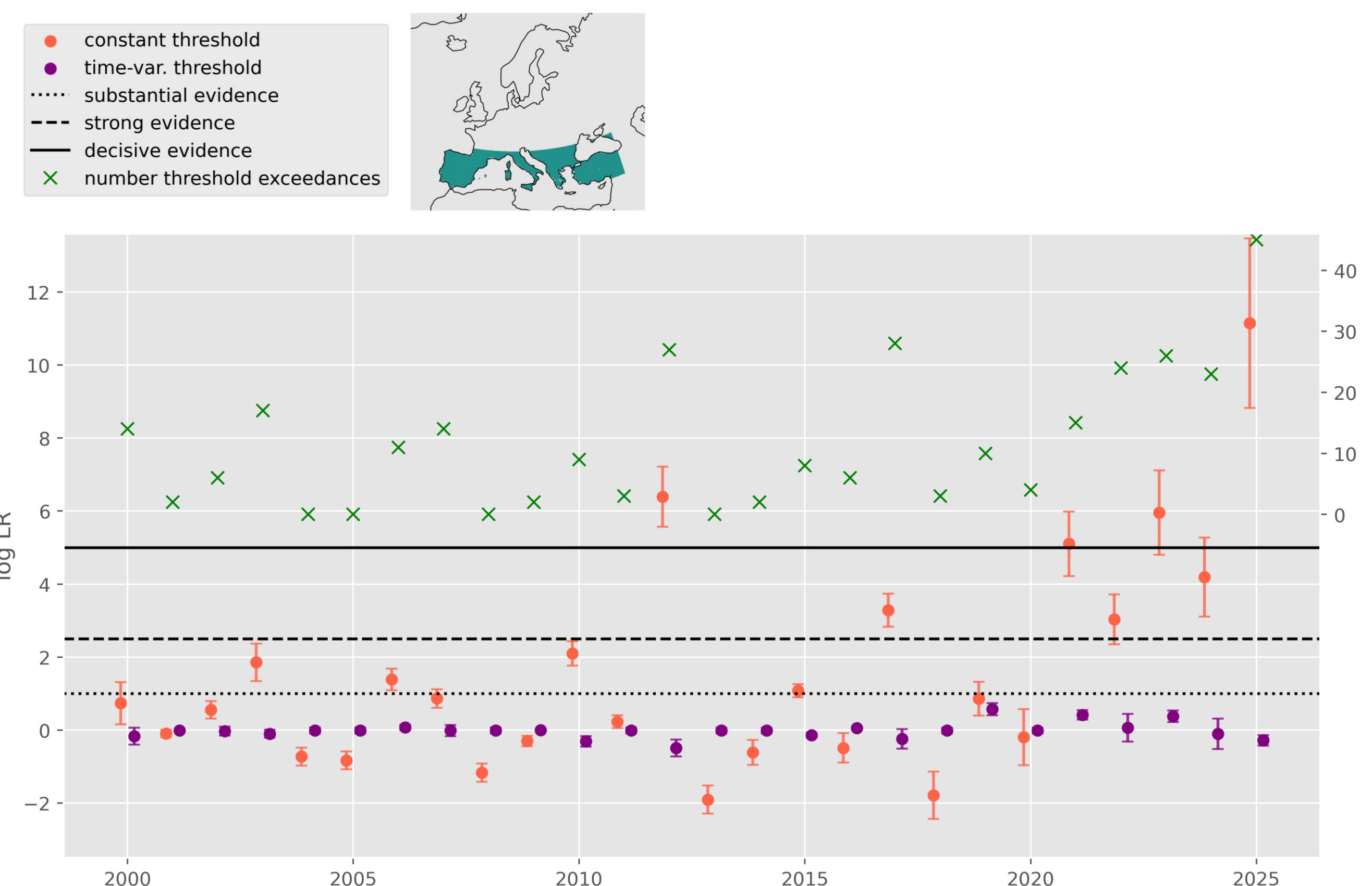
1. RQ: Constant threshold



2. RQ: Variable threshold



Resulting Likelihood ratios for southern Europe



Conclusion

- approach enables us to statistically model the temporal development of extreme events and thus also to conduct a corresponding attribution study
- Strong attribution result for heatwaves across all regions, especially in central and southern Europe

With funding from the:



References

- Cooley, D. and Thibaud, E.: Decompositions of dependence for high-dimensional extremes, *Biometrika*, 106, 587–604, <https://doi.org/10.1093/biomet/asz028>, 2019.
- Meurer, P., Buschow, S., Szemkus, S., & Friederichs, P.: Non-stationary time series attribution for heatwaves over Europe. *arXiv*. <https://arxiv.org/abs/2601.05841>, 2026.
- Szemkus, S. and Friederichs, P.: Spatial patterns and indices for heat waves and droughts over Europe using a decomposition of extremal dependency, *Advances in Statistical Climatology, Meteorology and Oceanography*, 10, 29–49, <https://doi.org/10.5194/asclmo-10-29-2024>, 2024.

